Relations and Functions

Case Study Based Questions

Case Study 1

In two different societies, there are some school going students-including girls as well as boys. Satish forms two sets with these students, as his college project.

Let A = (a_1, a_2, a_3, a_4, a_5} and B = {b_1, b_2, b_3, b_4} where a's and b's are the school going students of first and second society respectively.

Satish decides to explore these sets for various types of relations and functions.

Based on the above information, solve the following questions:

Q1. Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

a. 0 b. 2^5

c. 2^{10} d. 2^{20}

Q2. Let R: A → A, R = {(x, y): x and y are students of same sex}. Then relation R is:

a. reflexive only

b. reflexive and symmetric but not transitive

c. reflexive and transitive but not symmetric

d. an equivalence relation

Q3. Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

a. 1024 b. 2^{10} (15)

c. 2^{10} (31) d. 2^{10} (63)

Q4. Let R : A → B, R = {(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)}, then R is:

a. neither one-one nor onto

b. one-one but, not onto

c. only onto, but not one-one

d. not a function

Q5. To help Satish in his project, Rajat decides to form onto function from set A to B. How many such functions are possible?

a. 342

b. 240

c. 729

d. 1024

Solutions

1. Number of reflexive relations defined on a set of n elements = $2^{n(n-1)}$

Therefore, number of reflexive relations defined on

set A having 5 elements = $2^{5 \times 4} = 2^{20}$.

So, option (d) is correct.

2. As (x, x) R for all x ∈ A, when x is either boy or girl.

So, R is reflexive.

Let (x, y) ∈ R that is, x and y are of same sex.

That means, y and x are also of same sex.

This implies, (y, x) ∈ R.

So, R is symmetric.

Also let (x, y) ∈ R and (y, z) ∈ R.

That means, x and y are of same sex; y and z are same sex. Clearly, x and z will also be of same sex. That implies, (x, z) ∈ R.

So, R is transitive.

Therefore, R is equivalence relation.

So, option (d) is correct.

3. No. of Symmetric relations defined on a set of n elements

$$= 2^{\frac{n(n+1)}{2}}.$$

Therefore, number of symmetric relations defined on set A having 5 elements

$$=2^{\frac{5\times6}{2}}=2^{15}$$

Therefore, number of symmetric relations defined on set B having 4 elements

$$=2^{\frac{5\times4}{2}}=2^{10}$$

Hence, the required difference is 2^{15} - 2^{10} - $2^{10}(31)$.

So, option (c) is correct.

4. For the element a_1 e A, we have different images under R.

Note that, we have (a_1, b_1), $(a_1, b_2) \in R$.

So, R is not a function.

So, option (d) is correct.

5. If A and B are two sets having m and n elements respectively such that m ≥ n, then total number of onto functions from set A to set B is

$$=\sum_{r=0}^{n}(-1)^r\times{}^nC_r\times(n-r)^m.$$

Here, $n(A)=5$ *i.e.*, $m=5$ and $n(B)=4$ *i.e.*, $n=4$

So, the number of onto functions from set A to set B

$$=\sum_{r=0}^{4}(-1)^r\times{}^4C_r\times(4-r)^5$$

$$=(-1)^0\times{}^4C_0\times(4-0)^5+(-1)^1\times{}^4C_1\times(4-1)^5+(-1)^2\times{}^4C_2\times(4-2)^5+(-1)^3\times{}^4C_3\times(4-3)^5+(-1)^4\times{}^4C_4\times(4-4)^5$$

$$=1\times1\times(4)^5+(-1)\times4\times(3)^5+1\times6\times(2)^5+(-1)\times4\times1+1\times1\times0$$

$$=1024-972+192-4=240.$$

So, option (b) is correct.

Case Study 2

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1, 2, 3, 4, 5, 6}. Let A be the set of players while B be the set of all possible outcomes. i.e, A = {S, D}, B = {1,2,3,4,5,6}

Based on the given information, solve the following questions:

Q1. Let R : B → B be defined by R = {(x, y) : y is divisible by x} is:

a. reflexive and transitive but not symmetric

b. reflexive and symmetric and not transitive

c. not reflexive but symmetric and transitive

d. equivalence

Q 2. Raji wants to know the number of functions from A to B. How many number of functions are possible?

a. 6^2 b. 2^6

c. 6! d. 2^{12}

Q3. Let R be a relation on B defined by R = {(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)}. Then R is:

a. symmetric b. reflexive

c. transitive d. None of these

Q4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

a.6^2 b. 2^6

c. 6! d. 2^{12}

Q5. Let R : B → B be defined by R = {(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}, then R is:

a. symmetric b. reflexive and transitive

c. transitive and symmetric d. equivalence

Solutions

1. ∴ R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1,2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 4), (2, 6), (3, 6)}

For reflexive, we know that x is divisible by x for all x ∊ B.

∴ (x, x) ∊ R for all x ∊ R. So, R is reflexive.

For symmetry, we observe that 6 is divisible by 2.

This means that (2,6) ∊ R but (6, 2) ∉ R. So, R is not symmetric.

For transitivity, let (x, y) ∊ R and (y, z) ∊ R, then z is divisible by x.

⇒ (x, z) ∊ R

For example, 4 is divisible by 2, 2 is divisible by 1

So, 4 is divisible by 1. So, R is transitive.

So, option (a) is correct.

2. Here, n(A) = 2 and n(B) = 6

∴ Number of functions from A to B = $[n(B)]^{n(A)} = 6^2$.

So, option (a) is correct.

3. Here B = {1, 2, 3, 4, 5, 6) and R = {(1, 2), (2, 2), (1,3), (3,4), (3, 1), (4,3), (5,5)}

Since, (a, a) ∉ R, for every a ∊ {1, 2, 3, 4, 5, 6}

Therefore, R is not reflexive.

Now, since, (1,2) ∊ R but (2, 1) ∉ R.

Therefore, R is not symmetric.

Also, it is observed that (a, b), (b, c) ∊ R

⇒ (a, c) R. for any a, b, c ∊ {1, 2, 3, 4, 5, 6}

As (1, 3), (3, 4) ∊ R, but (1, 4) ∉ R

Therefore, R is not transitive.

So, option (d) is correct.

4. Here, n(A) = 2 and n(B) = 6

∴ Number of relations from A to B = $2^{n(A) \times n(B)}$

= $2^{2 \times 6} = 2^{12}$.

So, option (d) is correct.

5. Here, B = {1, 2, 3, 4, 5, 6} and R = {(1, 2), (2, 2), (1, 1), (3, 3), (4, 4), (5, 5), (6, 6)}

Since, (a, a) ∈ R for every a ∈ = {1, 2, 3, 4, 5, 6}

So, R is reflexive.

Now, since (1, 2) ∈ R but (2, 1) ∉ R.

So, R is not symmetric.

Also, it is observed that (a, b), (b, c) ∈ R

(a, c) ∈ R for any a, b, c ∈ = {1, 2, 3, 4, 5, 6}

So, R is transitive.

So, option (b) is correct.

Case Study 3

Archana visited the exhibition along with her family. The exhibition had a huge swing, which attracted many children. Archana found that the swing traced the path of a parabola as given by $y = x^2$.

Based on the above information, solve the following questions:

Q1. Let f : R→ R be defined by $f(x) = x^2$ is ....

a. neither surjective nor injective

b. surjective

c. injective

d. bijective

Q 2. Let f : N → N be defined by $f(x) = x^2$ is ....

a. surjective but not injective

b. surjective

c. injective

d. bijective

Q3. Let f : {1,2,3,...} {1, 4, 9,...} be defined by $f(x) = x^2$ is ....

a. bijective

b. surjective but not injective

c. injective but surjective

d. Neither surjective nor injective

Q4. Let N → R be defined by $f(x) = x^2$. Range of the function among the following is ....

a. {1, 4, 9, 16, ...)

b. {1, 4, 8, 9, 10,...}

c. {1, 4, 9, 15, 16, ...}

d. {1, 4, 8, 16, ...)

Q5. The function f : Z → Z defined by $f(x) = x^2$ is

a. neither injective nor surjective

b. injective

c. surjective

d. bijective

Solutions

1. f : R → R is given by $f(x) = x^2$.

It is seen that f(-1) = f(1) = 1 but -1+1

So, f is not injective.

Now, -2 ∈ R. But there does not exist any element

x ∈ R such that f(x) = x^2 = -2.

So, f is not surjective.

Hence, function is neither surjective nor injective.

So, option (a) is correct.

2. f : N → N is given by f(x) = x^2

It is seen that for x, y ∈ N, f(x) = f(y)

So, f is injective.

x^2 = y2

x = y

[∴ x and y are positive numbers]

Now, 2 ∈ N but there does not exist any x in N such that f(x) = x^2 = 2.

It means there is some element in co-domain in which do not have any images.
Therefore, f is not surjective.

So, option (c) is correct.

3. f : {1, 2, 3, ... } → {1, 4, 9, ...} is given by f(x) = x^2.

It is seen that for x_1, x_2 ∈ {1, 2, 3, ...}

$$f(x_1) = f(x_2)$$
$$\Rightarrow \quad x_1^2 = x_2^2$$
$$\Rightarrow \quad x_1 = x_2$$
$$[\because x_1 \text{ and } x_2 \text{ are positive numbers}]$$

So, f is injective.

Now, there exist any element x in {1, 2, 3, ...} such f(x) = x^2

e.g. At x = 1, f(1) = 1

At x = 2, f(2) = 4

At x = 3, f(3) = 9...

It means all elements in co-domain have images.

So, f is surjective.

Hence, f is bijective function.

So, option (a) is correct.

4. f : N ⟶ R is given by f(x) = x^2

At x=1, f(1) = 1

At x=2, f(2) = 4

At x=3, f(3) = 9 ......

So,

range of f(x) = {1, 4, 9, 16, ...}

So, option (a) is correct.

5. f: Z⟶ Z is given by f(x) = x^2

It is seen that f(-1) = f(1) = 1

but -1 ≠ 1.

So, f is not injective.

Now, -2 ∊ Z. But there does not exist any elements

X ∊ Z such that f(x) = x^2 = -2.

So, f is not surjective.

Hence, function f is neither injective nor surjective.

So, option (a) is correct.

Case Study 4

Consider the mapping ƒ : A ⟶ B is defined by

$f(x)=\dfrac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, solve the following questions:

Q1. Domain of ƒ is:

a. R - {2}   b. R

c. R - {1, 2}   d. R – {0}

Q2. Range of ƒ is:

a. R   b. R - {1}

c. R – {0}   d. R - {1,2}

Q3. If g : R - {2} → R - {1} is defined by g(x) = 2f(x) - 1, then g(x) in terms of x is:

a. $\frac{x+2}{x}$ b. $\frac{x+1}{x-2}$ c. $\frac{x-2}{x}$ d. $\frac{x}{x-2}$

Q4. The function g defined above, is:

a. one-one   b. many-one

c. into   d. None of these

Q5. A function f(x) is said to be one-one iff:

a. $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$

b. $f(-x_1) = f(x_2) \Rightarrow -x_1 = x_2$

c. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

d. None of the above

Solutions

1. For f(x) to be defined x - 2 ≠ 0 i.e., x ≠ 2

∴ Domain of f = R – {2}

So, option (a) is correct.

2. Let   y = f(x), then y

$$= \frac{x-1}{x-2}$$

⇒ xy - 2y = x - 1

⇒ xy - x - 2y – 1

$$\Rightarrow \qquad x = \frac{2y-1}{y-1}$$

Since, $x \in R - \{2\}$
therefore $y \neq 1$
Hence, range of $f = R - \{1\}$
So, option (b) is correct.

3. We have, $g(x) = 2f(x) - 1$

$$= 2\left(\frac{x-1}{x-2}\right) - 1$$

$$= \frac{2x - 2 - x + 2}{x-2} = \frac{x}{x-2}$$

So, option (d) is correct.

4. We have, $g(x) = \frac{x}{x-2}$

Let $g(x_1) = g(x_2)$

$\Rightarrow \frac{x_1}{x_1 - 2} = \frac{x_2}{x_2 - 2}$

$\Rightarrow x_1x_2 - 2x_1 = x_1x_2 - 2x_2$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Thus, $g(x_1) = g(x_2)$

$\Rightarrow x_1 = x_2$

Hence, $g(x)$ is one-one.

Also, range of $g(x)$ = co-domain

So, $g(x)$ is onto.

So, option (a) is correct.

5. $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

So, option (c) is correct.

Case Study 5

Students of Grade 12, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line y = x - 4. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Based on the above information, solve the following questions:

Q 1. Let relation R be defined by $R = \{(L_1, L_2) : L_1 || L_2$ where $L_1, L_2 \in L\}$ then show that the relation R is an equivalence relation.

Q 2. Let $R = \{(L_1, L_2) : L_1 \perp L_2$ where $L_1, L_2 \in L\}$, then show that R is only symmetric relation.

Q 3. Show that the function $f : R \to R$ defined by $f(x) = x - 4$ is bijective.

Solutions

1. Here, $R = \{(L_1, L_2) : L_1 || L_2$ where $L_1, L_2 \in L\}$

R is reflexive as any line L_1 is parallel to itself

i.e., $(L_1, L_1) \in R$

Now, let $(L_1, L_2) \in R$

$\Rightarrow$ L_1 is parallel to $L_2 \Rightarrow L_2$ is parallel to L_1

$\Rightarrow$ $(L_2, L_1) \in R$

So, R is symmetric.

Now, let $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow$ L_1 is parallel to L_2, also L_2 is parallel to L_3

$\Rightarrow$ L_1 is parallel to L_3

$\Rightarrow$ $(L_1, L_3) \in R$. So, R is transitive.

Hence, R is an equivalence relation.

2. Here, $R = \{(L_1, L_2): L_1 \perp L_2$ where $L_1, L_2 \in L\}$

R is not reflexive as any line L_1 is not perpendicular to itself.

i.e., $(L_1, L_1) \notin R$

Now, let $(L_1, L_2) \in R$

$\Rightarrow$ L_1 is perpendicular to L_2

$\Rightarrow$ L_2 is perpendicular to L_1

$\Rightarrow$ $(L_2, L_1) \in R$

So, R is symmetric.

Now, let $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow$ L_1 is perpendicular L_2, also L_2 is perpendicular to L_3

$\Rightarrow$ L_1 is parallel to L_3 *i.e.*, L_1 is not perpendicular to L_3.

$\Rightarrow$ $(L_1, L_3) \notin R$. So, R is not transitive.

3. Here, $f : R \to R$ is defined by $f(x) = x - 4$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow$ $x_1 - 4 = x_2 - 4$

$\Rightarrow$ $x_1 = x_2$

Therefore, f is one-one.

For any real number y in R, there exist $(y + 4)$ in R such that

$$f(y + 4) = (y + 4) - 4 = y$$

So, f is onto. Hence, f is bijective.

Case Study 6

An organization conducted bike race under two different categories Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let B = $\{b_1, b_2, b_3\}$ and G = $\{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.

Based on the above information, solve the following questions: (CBSE 2023)

Q1. How many relations are possible from B to G?

Q2. Among all the possible relations from B to G, how many functions can be formed from B to G?

Q3. Let R : B → B be defined by R = {(x, y) : x and y are students of the same sex}. Check whether R is an equivalence relation.

Or

A function f : B → G be defined by f = {(b_1, g_1), (b_2, g_2), (b_3, g_1)). Check whether f is objective. Justify your answer.

Solutions

1. The number of relations from B to G is

$2^{n(B) \times n(G)} = 2^{3 \times 2}$

$=2^6 = 64$

2. The number of functions from B to G is

$(n(G)]^{n(B)}$ i.e., 2^3 or 8.

3. Reflexive

Since x and x are of the same sex.

So, (x, x) ∈ R for all x.

∴ R is reflexive.

Symmetric

If x and y are of the same sex. Then y and x are of the same sex.

$i.e. (x, y) \in R \Rightarrow (y, x) \in R \ \forall x, y$

So, R is symmetric.

Transitive

If x and y are of the same sex; y and z are of the same sex, then x and z are of the same sex.

So, R is transitive

Hence, R is an equivalence relation.

Or

Given function f : B → G such that

f = {(b_1, g_1), (b2, g2), (b3, g1)}

Here we see that b_1 and b3 have same image g_1, so it is not one to one function.

Thus, f(x) is not bijective function.

Case Study 7

A relation R on a set A is said to be an equivalence relation on A iff it is:

Reflexive *i.e.*, $(a,a) \in R \ \forall \ a \in A$.
Symmetric *i.e.*, $(a,b) \in R \Rightarrow (b,a) \in R$ for any $a,b \in A$.
Transitive *i.e.*, $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow (a,c) \in R$ for any $a,b,c \in A$.

Based on the above information, solve the following questions:

Q1. If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A = {1, 2, 3}, then show that the relation R is only reflexive.

Q 2. If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then show that relation R is only symmetric.

Q3. If the relation Ron the set N of all natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}, then show that R is not reflexive as well as symmetric but R is transitive.

Q4. If the relation R on the set A = {1, 2, 3, ……, 13, 14} defined as R = {(x, y) : 3x − y = 0}, then show that R is not reflexive, symmetric and transitive.

Q5. If the relation R on the set A = {1, 2, 3} defined as R = {(1, 1), (1,2), (1,3), (2, 1), (2,2), (2,3), (3, 1), (3,2), (3, 3)}, then show that Ris an equivalence relation.

Solutions

1. Clearly, (1, 1), (2, 2), (3, 3), $\in R$. So, R is reflexive on A.
Since, (1, 2) $\in R$ but (2, 1) $\notin R$. So, R is not symmetric on A.
Since, (2, 3), $\in R$ and (3, 1) $\in R$ but (2, 1) $\notin R$. So, R is not transitive on A.

2. Since, (1, 1), (2, 2) and (3, 3) are not in R.
So, R is not reflexive on A.
Now, (1, 2) $\in R \Rightarrow$ (2, 1) $\in R$
and (1, 3) $\in R \Rightarrow$ (3, 1) $\in R$.
So, R is symmetric
Clearly, (1, 2) $\in R$ and (2, 1) $\in R$ but (1, 1) $\notin R$.
So, R is not transitive on A.

3. We have, $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$,
where $x, y \in N$.

$\therefore \quad R = \{(1, 6), (2, 7), (3, 8)\}$

Clearly, (1, 1), (2, 2) etc. are not in R. So, R is not reflexive.
Since, $(1, 6) \in R$ but $(6, 1) \notin R$. So, R is not symmetric.

Since, $(1, 6) \in R$ and there is no order pair in R which has 6 as the first element. Same is the case for (2, 7) and (3, 8).

So, R is transitive as transitivity is not contradicted.

4. We have, $R = \{(x, y) : 3x - y = 0\}$,
where $x, y \in A = \{1, 2, \ldots\ldots, 14\}$

$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Clearly, $(1, 1) \notin R$. So, R is not reflexive on A.
Since, $(1, 3) \in R$ but $(3, 1) \notin R$. So, R is not symmetric on A.
Since, $(1, 3) \in R$ and $(3, 9) \in R$ but $(1, 9) \notin R$. So, R is not transitive on A.

5. Clearly, $(1, 1), (2, 2), (3, 3) \in R$. So, R is reflexive on A.

We find that the ordered pairs obtained by interchanging the components of ordered pairs in R are also in R. So, R is symmetric on A.

For $1, 2, 3 \in A$ such that (1, 2) and (2, 3) are in R implies that (1, 3) is also, in R. So, R is transitive on A.
Thus, R is an equivalence relation.

Solutions for Questions 8 to 9 are Given Below

Case Study 8

A relation R on a set A is said to be an equivalence relation on A iff it is

- Reflexive *i.e.*, $(a, a) \in R \ \forall \ a \in A$.
- Symmetric *i.e.*, $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b \in A$.
- Transitive *i.e.*, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$.

Based on the above information, answer the following questions.

(i) If the relation $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ defined on the set $A = \{1, 2, 3\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(ii) If the relation $R = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$ defined on the set $A = \{1, 2, 3\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iii) If the relation R on the set N of all natural numbers defined as $R = \{(x, y) : y = x + 5$ and $x < 4\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) equivalence

(iv) If the relation R on the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) None of these

(v) If the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$, then R is

(a) reflexive only (b) symmetric only

(c) transitive only (d) equivalence

Case Study 9

Consider the mapping $f : A \to B$ is defined by $f(x) = \dfrac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, answer the following questions.

(i) Domain of f is

(a) $R - \{2\}$ (b) R (c) $R - \{1, 2\}$ (d) $R - \{0\}$

(ii) Range of f is

(a) R (b) $R - \{1\}$ (c) $R - \{0\}$ (d) $R - \{1, 2\}$

(iii) If $g : R - \{2\} \to R - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is

(a) $\frac{x+2}{x}$ (b) $\frac{x+1}{x-2}$ (c) $\frac{x-2}{x}$ (d) $\frac{x}{x-2}$

(iv) The function g defined above, is

(a) One-one (b) Many-one (c) into (d) None of these

(v) A function $f(x)$ is said to be one-one iff

(a) $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$ (b) $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$

(c) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ (d) None of these

HINTS & EXPLANATIONS

8. (i) (a): Clearly, (1, 1), (2, 2), (3, 3), $\in R$. So, R is reflexive on A.

Since, $(1, 2) \in R$ but $(2, 1) \notin R$. So, R is not symmetric on A.

Since, $(2, 3), \in R$ and $(3, 1) \in R$ but $(2, 1) \notin R$. So, R is not transitive on A.

(ii) (b): Since, (1, 1), (2, 2) and (3, 3) are not in R.

So, R is not reflexive on A.

Now, $(1, 2) \in R \Rightarrow (2, 1) \in R$

and $(1, 3) \in R \Rightarrow (3, 1) \in R$.

So, R is symmetric

Clearly, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

So, R is not transitive on A.

(iii) (c): We have, $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$, where $x, y \in N$.

$\therefore \quad R = \{(1, 6), (2, 7), (3, 8)\}$

Clearly, (1, 1), (2, 2) etc. are not in R. So, R is not reflexive.

Since, $(1, 6) \in R$ but $(6, 1) \notin R$. So, R is not symmetric.

Since, $(1, 6) \in R$ and there is no order pair in R which has 6 as the first element. Same is the case for (2, 7) and (3, 8).

So, R is transitive.

(iv) (d): We have, $R = \{(x, y) : 3x - y = 0\}$, where $x, y \in A = \{1, 2, ..., 14\}$

$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

Clearly, $(1, 1) \notin R$. So, R is not reflexive on A.

Since, $(1, 3) \in R$ but $(3, 1) \notin R$. So, R is not symmetric on A.

Since, $(1, 3) \in R$ and $(3, 9) \in R$ but $(1, 9) \notin R$. So, R is not transitive on A.

(v) (d): Clearly, (1, 1), (2, 2), (3, 3) $\in R$. So, R is reflexive on A.

We find that the ordered pairs obtained by interchanging the components of ordered pairs in R are also in R. So, R is symmetric on A.

For 1, 2, 3 $\in A$ such that (1, 2) and (2, 3) are in R implies that (1, 3) is also, in R. So, R is transitive on A.

Thus, R is an equivalence relation.

9. (i) (a): For $f(x)$ to be defined $x - 2 \neq 0$ *i.e.*, $x \neq 2$

$\therefore$ Domain of $f = R - \{2\}$

(ii) (b): Let $y = f(x)$, then $y = \frac{x-1}{x-2}$

$\Rightarrow xy - 2y = x - 1 \Rightarrow xy - x = 2y - 1 \Rightarrow x = \frac{2y-1}{y-1}$

Since, $x \in R - \{2\}$, therefore $y \neq 1$

Hence, range of $f = R - \{1\}$

(iii) (d): We have, $g(x) = 2f(x) - 1$

$= 2\left(\frac{x-1}{x-2}\right) - 1 = \frac{2x-2-x+2}{x-2} = \frac{x}{x-2}$

(iv) (a): We have, $g(x) = \frac{x}{x-2}$

Let $g(x_1) = g(x_2) \Rightarrow \frac{x_1}{x_1-2} = \frac{x_2}{x_2-2}$

$\Rightarrow x_1x_2 - 2x_1 = x_1x_2 - 2x_2 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

Thus, $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$

Hence, $g(x)$ is one-one.

(v) (c)